PARABOLA

ESERCIZIO GUIDA:

$$y = x^2 - 5x + 6$$

$a = 1$
$b = -5$
$c = 6$

$$\Delta = b^2 - 4ac = (-5)^2 - 4 \cdot 1 \cdot 6 = 25 - 24 = 1$$

VERTICE: $V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) \rightarrow V\left(\frac{5}{2}, -\frac{1}{4}\right)$

FUOCO: $F\left(-\frac{b}{2a}, \frac{1-\Delta}{4a}\right) \rightarrow F\left(\frac{5}{2}, 0\right)$

DIRETTRICE: $y = -\frac{1+\Delta}{4a} \rightarrow y = -\frac{2}{4} \rightarrow y = -\frac{1}{2}$

ASSE DI SIMMETRIA: $x = -\frac{b}{2a} \rightarrow x = \frac{5}{2}$

INTERSEZIONI ASSE X:

$$\begin{cases} y = 0 \\ y = x^2 - 5x + 6 \end{cases} \rightarrow x^2 - 5x + 6 = 0$$

$\Delta = 1$

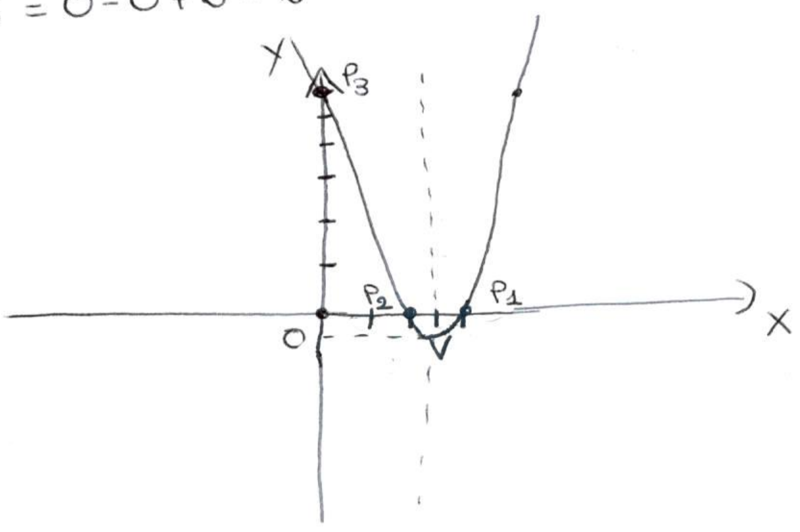
$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{5 \pm 1}{2} \begin{cases} \frac{6}{2} = 3 = x_1 \\ \frac{4}{2} = 2 = x_2 \end{cases}$$

$\rightarrow P_1(3, 0)$; $P_2(2, 0)$

INTERSEZIONI ASSE Y:

$$\begin{cases} x = 0 \\ y = x^2 - 5x + 6 = 0 - 0 + 6 = 6 \end{cases} \rightarrow P_3(0, 6)$$

DISEGNO:



INTERSEZIONI RETTA PARABOLA

ESERCIZIO GUIDA:

TROVARE GLI EVENTUALI PUNTI DI INTERSEZIONE TRA LA RETTA r DI EQUAZIONE: $y = 4x + 1$ E LA PARABOLA γ DI EQUAZIONE: $y = 2x^2 + x + 2$

METTO A SISTEMA RETTA E PARABOLA:

1° METTO A SISTEMA RETTA E PARABOLA:

$$\begin{cases} y = 4x + 1 \\ y = 2x^2 + x + 2 \end{cases}$$

2° METODO DI SOSTITUZIONE

$$\begin{cases} y = 4x + 1 \\ 4x + 1 = 2x^2 + x + 2 \end{cases} \rightarrow 2x^2 - 3x + 1 = 0$$

$a = 2$, $b = -3$, $c = 1$ $\quad \Delta = b^2 - 4 \cdot a \cdot c = 9 - 8 = 1 > 0$

$$\rightarrow x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{3 \pm 1}{4} \begin{cases} 1 = x_1 \\ \frac{1}{2} = x_2 \end{cases}$$

$$\begin{cases} x_1 = 1 \\ y = 4 \cdot 1 + 1 = 5 \end{cases} \cup \begin{cases} x_2 = \frac{1}{2} \\ y = 4 \cdot \frac{1}{2} + 1 = 3 \end{cases}$$

$\rightarrow$ 2 PUNTI DI INTERSEZIONE: $P_1(1, 5)$ e $P_2(\frac{1}{2}, 3)$

IN GENERALE:

$\Delta > 0$: LA RETTA È SECANTE E INTERSECA LA PARABOLA IN 2 PUNTI

$\Delta = 0$: " TANGENTE " 1 PUNTO

$\Delta < 0$: " ESTERNA: NON INTERSECA LA PARABOLA.

PER CASA:

1. TROVARE FUOCO, VERTICE, DIRETTRICE, ASSE DI SIMMETRIA, INTERSEZIONI CON GLI ASSI E DISEGNARE LE SEGUENTI PARABOLE:
   - (a) $y = x^2 - 8x + 8$
   - (b) $y = -x^2 + 4x - 3$
2. TROVARE LE INTERSEZIONI TRA LA RETTA r: $y = -2x + 6$ E LA PARABOLA γ: $y = -2x^2 + 2x + 4$. DISEGNARE.
3. TROVARE LE INTERSEZIONI TRA LA RETTA r: $y = 5$ E LA PARABOLA $y = -2x^2 + 2x + 4$. DISEGNARE.

TROVARE L'EQUAZIONE DELLA PARABOLA DATI IL VERTICE E IL FUOCO:

ESEMPIO GUIDA:
TROVARE L'EQ. DELLA PARABOLA CHE HA VERTICE

$V\underset{(x_V,\,y_V)}{(1,3)}$ E FUOCO $\underset{(x_F,\,y_F)}{(1,2)}$

$$y = ax^2 + bx + c$$

$$V\left(\overset{x_V}{-\frac{b}{2a}}, \overset{y_V}{-\frac{\Delta}{4a}}\right) \qquad \Delta = b^2 - 4ac$$

$$F\left(\overset{x_F}{-\frac{b}{2a}}, \overset{y_F}{\frac{1-\Delta}{4a}}\right)$$

VADO A IMPORRE:

$$\begin{aligned} &(1):\ x_V = 1 \\ &(2):\ y_V = 3 \\ &(3):\ y_F = 2 \end{aligned} \quad \begin{cases} -\frac{b}{2a} = 1 \to b = -2a \\ -\frac{\Delta}{4a} = 3 \to +\frac{b^2-4ac}{4a} = -3 \\ \frac{1-\Delta}{4a} = 2 \to \frac{1-b^2+4ac}{4a} = 2 \end{cases}$$

RISOLVO IL SISTEMA COL METODO DI SOSTITUZIONE:

$$\begin{cases} b = -2a \\ \frac{(-2a)^2-4ac}{4a} = -3 \to \frac{4a^2-4ac}{4a} = -3 \to a - c = -3 \to c = a+3 \\ \frac{1-(-2a)^2+4a\cdot(a+3)}{4a} = 2 \end{cases}$$

$$\begin{cases} b = -2a \\ c = a+3 \\ \frac{1-\cancel{4a^2}+\cancel{4a^2}+12a}{4a} = \frac{2}{1} \to 1+12a = 8a \\ \qquad \to 4a = -1 \to a = -\frac{1}{4} \end{cases}$$

$$\begin{cases} a = -\frac{1}{4} \\ b = -2\cdot\left(-\frac{1}{4}\right) = +\frac{1}{2} \\ c = -\frac{1}{4}+3 = \frac{-1+12}{4} = +\frac{11}{4} \end{cases}$$

$$y = -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4}$$

DISEGNO LA PARABOLA:

INTERSEZIONI ASSE X

$$\begin{cases} y = 0 \\ y = -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4} \end{cases} \Rightarrow -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4} = 0$$

$$\Rightarrow \frac{1}{4}x^2 - \frac{1}{2}x - \frac{11}{4} = 0$$

$$\frac{x^2 - 2x - 11}{\cancel{4}} = 0$$

$$x^2 - 2x - 11 = 0$$

$a = 1$, $b = -2$, $c = -11$

$$\Delta = b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot (-11) = 4 + 44 = 48$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2} = 1 \pm 2\sqrt{3}$$

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$

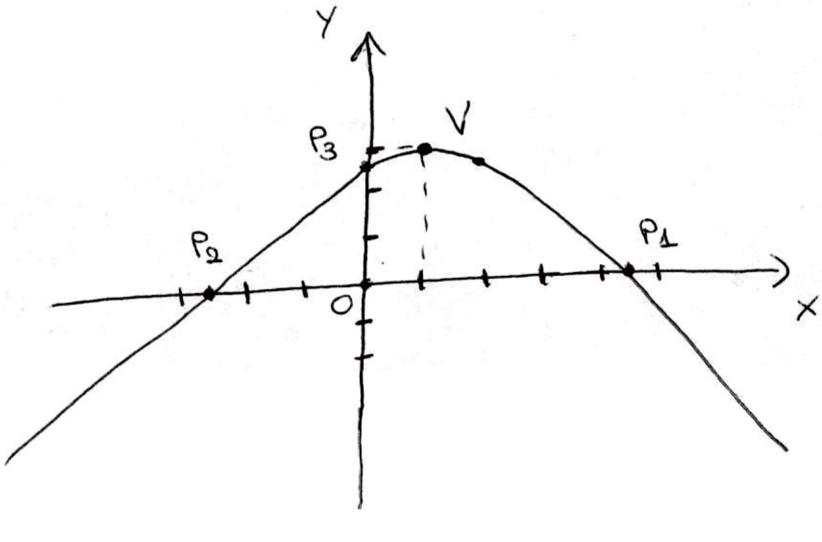
$\rightarrow$ 2 INTERSEZIONI:

$P_1(1+2\sqrt{3}, 0)$ $\quad$ $P_2(1-2\sqrt{3}, 0)$

INTERSEZIONI ASSE Y:

$$\begin{cases} x = 0 \\ y = -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4} = -\frac{1}{4} \cdot 0 + \frac{1}{2} \cdot 0 + \frac{11}{4} = +\frac{11}{4} \end{cases}$$

$\rightarrow P_3\left(0, -\frac{11}{4}\right)$



ESEMPIO GUIDA:
TROVARE L'EQ. DELLA PARABOLA DATO IL VERTICE
$V(2,4)$ E LA DIRETTRICE $y = 3$

$$y = ax^2 + bx + c$$

$$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) \qquad \Delta = b^2 - 4ac$$

$$y = -\frac{1+\Delta}{4a}$$

IMPONGO LE CONDIZIONI:

(1): $x_V = -\frac{b}{2a} = 2$

(2): $y_V = -\frac{\Delta}{4a} = 4$

(3): $-\frac{1+\Delta}{4a} = 3$

$$\begin{cases} -\frac{b}{2a} = 2 \rightarrow b = -4a \\ -\frac{\Delta}{4a} = 4 \rightarrow \frac{\Delta}{4a} = -4 \rightarrow \frac{b^2-4ac}{4a} = -4 \\ -\frac{1+\Delta}{4a} = 3 \rightarrow \frac{1+\Delta}{4a} = -3 \rightarrow \frac{1+b^2-4ac}{4a} = -3 \end{cases}$$

RISOLVO COL METODO DI SOSTITUZIONE:

$$\begin{cases} b = -4a \\ \frac{(-4a)^2-4ac}{4a} = -4 \rightarrow \frac{16a^2-4ac}{4a} = -4 \rightarrow 4a - c = -4 \rightarrow c = 4a+4 \\ \frac{1+(-4a)^2-4a\cdot(4a+4)}{4a} = -3 \end{cases}$$

$$\rightarrow \frac{1+16\cancel{a^2}-16\cancel{a^2}-16a}{4a} = -\frac{3}{1} \rightarrow \frac{1-16a}{\cancel{4a}} = \frac{-12a}{\cancel{4a}}$$

$$\rightarrow -16a + 12a = -1$$

$$-4a = -1$$

$$a = \frac{1}{4}$$

$$\begin{cases} a = \frac{1}{4} \\ b = -4 \cdot \frac{1}{4} = -1 \\ c = \cancel{4} \cdot \frac{1}{\cancel{4}} + 4 = 1 + 4 = 5 \end{cases}$$

$y = \frac{1}{4}x^2 - x + 5$

DISEGNO LA PARABOLA:

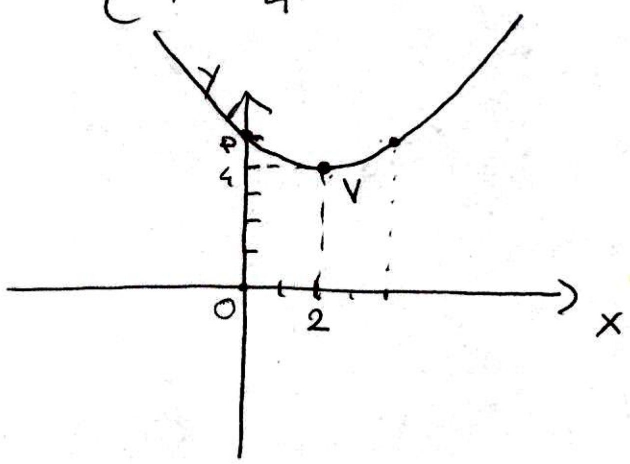
INT. ASSE X:

$$\begin{cases} y = 0 \\ \frac{1}{4}x^2 - x + 5 = 0 \end{cases} \rightarrow x^2 - 4x + 20 = 0$$

$\Delta = (-4)^2 - 4 \cdot 20 = 16 - 80 = -64 < 0$ la parabola non incontra l'asse x

INT. ASSE Y:

$$\begin{cases} x = 0 \\ y = \frac{1}{4} \cdot 0 - 0 + 5 = 5 \end{cases} \rightarrow P(0,5)$$



ESERCIZI:

TROVARE L'EQ. DELLA PARABOLA DATI:

1. $V(1,2)$ e $F(1,3)$
2. $V(-2,1)$ e $F(-2,2)$
3. $V(1,1)$ e $y = 2$
4. $V(-1,-2)$ e $y = -3$

FARE I DISEGNI

CORREZIONE ESERCIZI ASSEGNATI:

① $y = ax^2 + bx + c$

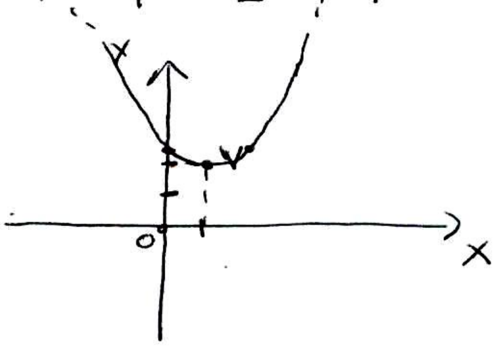
$V(1,2)$ $\quad F(1,3)$

$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$ $\quad F\left(-\frac{b}{2a}, \frac{1-\Delta}{4a}\right)$

$$\begin{cases} -\frac{b}{2a} = 1 \rightarrow b = -2a \\ -\frac{\Delta}{4a} = 2 \rightarrow \frac{\Delta}{4a} = -2 \rightarrow \frac{(-2a)^2 - 4ac}{4a} = -2 \rightarrow \frac{4a^2 - 4ac}{4a} = -2 \rightarrow a - c = -2 \rightarrow c = a + 2 \\ \frac{1-\Delta}{4a} = 3 \end{cases}$$

$$\begin{cases} b = -2a \\ c = a + 2 \\ \frac{1 - (-2a)^2 + 4a(a+2)}{4a} = 3 \rightarrow 1 - \cancel{4a^2} + \cancel{4a^2} + 8a = 12a \rightarrow -4a = -1 \rightarrow a = \frac{1}{4} \end{cases}$$

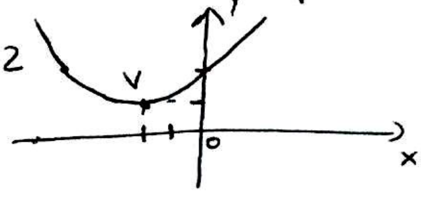
$$\begin{cases} a = \frac{1}{4} \\ b = -2 \cdot \frac{1}{4} = -\frac{1}{2} \\ c = \frac{1}{4} + 2 = \frac{9}{4} \end{cases} \Rightarrow y = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{9}{4}$$



②
$$\begin{cases} -\frac{b}{2a} = -2 \rightarrow b = 4a \\ -\frac{\Delta}{4a} = 1 \rightarrow \frac{\Delta}{4a} = -1 \Rightarrow \frac{(4a)^2 - 4a \cdot c}{4a} = -1 \Rightarrow 4a - c = -1 \Rightarrow c = 4a + 1 \\ \frac{1-\Delta}{4a} = 2 \rightarrow \frac{1 - (4a)^2 + 4a(4a+1)}{4a} = 2 \Rightarrow \end{cases}$$

$$\rightarrow 1 - \cancel{16a^2} + \cancel{16a^2} + 4a = 8a \rightarrow -4a = -1 \rightarrow a = \frac{1}{4}$$

$$\begin{cases} a = \frac{1}{4} \\ b = 4 \cdot \frac{1}{4} = 1 \\ c = \cancel{4} \cdot \frac{1}{\cancel{4}} + 1 = 1 + 1 = 2 \end{cases} \qquad y = \frac{1}{4}x^2 + x + 2$$



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(3) $y = ax^2 + bx + c$

$V(1, 1)$ $\qquad y = 2$

$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$ $\qquad y = -\frac{1+\Delta}{4a}$

$$\begin{cases} -\frac{b}{2a} = 1 \rightarrow b = -2a \\ -\frac{\Delta}{4a} = 1 \rightarrow \frac{\Delta}{4a} = -1 \rightarrow \frac{b^2-4ac}{4a} = -1 \rightarrow \frac{(-2a)^2-4ac}{4a} = -1 \rightarrow \\ \qquad \rightarrow \frac{4a^2-4ac}{4a} = -1 \rightarrow \\ -\frac{1+\Delta}{4a} = 2 \rightarrow \frac{1+\Delta}{4a} = -2 \qquad \rightarrow a - c = -1 \rightarrow c = a+1 \end{cases}$$

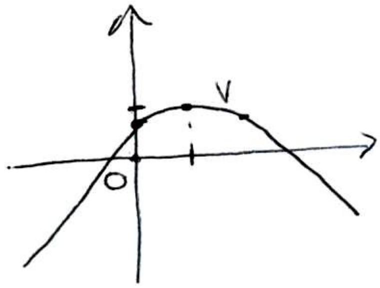
$$\begin{cases} b = -2a \\ c = a+1 \\ \frac{1+b^2-4ac}{4a} = -2 \rightarrow \frac{1+(-2a)^2-4a\cdot(a+1)}{\cancel{4a}} = -\frac{8a}{\cancel{4a}} \rightarrow \end{cases}$$

$$\rightarrow 1 + \cancel{4a^2} - \cancel{4a^2} - 4a = -8a$$

$$\rightarrow -4a + 8a = -1$$

$$\rightarrow \frac{4a}{4} = -\frac{1}{4} \rightarrow a = -\frac{1}{4}$$

$$\begin{cases} a = -\frac{1}{4} \\ b = -2\cdot\left(-\frac{1}{4}\right) = \frac{1}{2} \\ c = -\frac{1}{4} + \frac{1}{1} = \frac{-1+4}{4} = \frac{3}{4} \end{cases} \rightarrow y = -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{3}{4}$$



(4)
$$\begin{cases} -\frac{b}{2a} = -1 \rightarrow b = 2a \\ -\frac{\Delta}{4a} = -2 \rightarrow \frac{\Delta}{4a} = 2 \rightarrow \frac{b^2-4ac}{4a} = 2 \\ -\frac{1+\Delta}{4a} = -3 \rightarrow \frac{1+\Delta}{4a} = 3 \rightarrow \frac{1+b^2-4ac}{4a} = 3 \end{cases}$$

$$\begin{cases} b = 2a \\ \frac{4a^2-4ac}{4a} = 2 \rightarrow a - c = 2 \rightarrow c = a-2 \\ \frac{1+4a^2-4a\cdot(a-2)}{\cancel{4a}} = \frac{12a}{\cancel{4a}} \rightarrow 1 + \cancel{4a^2} - \cancel{4a^2} + 8a = 12a \rightarrow \end{cases}$$

$$a = \frac{1}{4}$$

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$$\begin{cases} a = \frac{1}{4} \\ b = 2 \cdot \frac{1}{4} = \frac{1}{2} \\ c = \frac{1}{4} - 2 = \frac{1-8}{4} = -\frac{7}{4} \end{cases}$$

$\rightarrow \quad y = \frac{1}{4}x^2 + \frac{1}{2}x - \frac{7}{4}$